SPATIAL FLOW REGIMES AROUND A CURVED SURFACE

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The computation of the flow and aerodynamic heating of different spatial local roughnesses on a body surface is an urgent scientific engineering problem (see, e.g., [1]). Analysis of the flows around roughnesses on a flat surface [2] disclosed an interaction mechanism between the spatial perturbed domains and the external inviscid flow, permitted the formulation of a whole series of original boundary value problems, investigation of the fundamental properties of their solutions and the construction of a classification scheme for such flow regimes.

Taking account of the body surface curvature is important in practice. This will afford the possibility of modelling the flow around roughnesses on a wing surface, on the wall of a curved channel, or on a turbine blade. Study of the laminary boundary layer flow interaction around a curved surface with a small roughness thereon disclosed the origin of a special modification of the longitudinal—transverse interaction theory in this case [3, 4]. A detailed bibliography on the asymptotic theory of interaction between spatial perturbed flow domains and an external inviscid flow is presented in [2, 3].

Systematic investigations of the viscous incompressible fluid flow around small spatial roughnesses on a curved surface are performed in this paper. It is obtained that the influence of surface curvature is felt only on roughnesses extended in the stream direction. A classification scheme for the flow regimes around such roughnesses is constructed, the differences in the interaction mechanisms between perturbed flow domains and an external inviscid stream are shown, and an explanation is proposed for the origination of perturbation transmission upstream during the flow around roughnesses on convex surfaces.

1. The flow around a curved plate (with a constant radius of curvature R) by a viscous fluid flow is considered for large but subcritical Reynolds numbers. It is assumed that a small spatial convexity or dent (Fig. 1) is found on a plate surface at a distance $L \leqslant R$ from the leading edge. A stationary solution is constructed for the Navier—Stokes equations for the spatial perturbed laminar flow domain as $Re = u_0 L/\nu = \varepsilon^{-2}$ tends to infinity (u_0 is the longitudinal velocity component in the external inviscid flow at the point where the small roughness is, and ν is the kinematic viscosity coefficient). Henceforth only dimensionless variables are used, where all the linear dimensions are referred to L, the velocity components to u_0, and the pressure to ρu_0^2 (ρ is the fluid density).

With respect to the size of the small roughness it is assumed that its characteristic thickness a is of the order of magnitude less than or equal to the characteristic thickness of the unperturbed boundary layer on a curved plate at this site ($a \leqslant \delta \sim O(\varepsilon)$), while its characteristic extent b is of an order of magnitude greater than or equal to a and less than or equal to one ($a \leqslant b \leqslant 1$). The characteristic width of the roughness c can be greater than or equal to a in order of magnitude $a (c \geqslant a)$. For $a \sim b$ or $a \sim c$, only the longitudinal or transverse dimension of the perturbed flow domain will be determined by the value of a. It is still evident that the size of this domain should be greater than the characteristic mean free path of the fluid molecules (a, b, c > ε^2), i.e., a, b, and c satisfy the relationships

$$\varepsilon^2 < a \leqslant \varepsilon,\ a \leqslant b \leqslant 1,\ a \leqslant c. \tag{1.1}$$

A general scheme was constructed in [2] for the flow regimes around roughnesses with the characteristic dimensions (1.1) on the surface of a flat plate. Cases were examined when viscous nonlinear perturbations, $u \sim \Delta u \sim \Delta p_1^{1/2}$ or $w \sim \Delta w \sim \Delta p_1^{1/2}$, were initiated near the roughness surface (u, w are the longitudinal and transverse velocity components, Δu, Δw are the perturbations of these components, and Δp_1 is the pressure perturbation induced by the small roughness on the flat plate surface). The pressure perturbation $\Delta p_2 \sim \pm k u^2 \Delta y$ is

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additionally induced during the flow around roughness on a curved plate because of the action of centrifugal forces (the plate curvature is $k = L/R = \varkappa K \leqslant 1$, $\varkappa \leqslant 1$, $K \sim O(1)$, Δy is the fluid layer thickness in which the pressure perturbation Δp_2 is induced, and the upper or lower sign refers to a convex or concave plate). Flow regimes around roughnesses on curved surfaces are investigated below when they induce viscous nonlinear perturbations and are identical in order of magnitude to the perturbations Δp_1 and Δp_2. Neither inessential nor passive terms describing the variation along the curved surface were taken into account in the determination of the orders of magnitude of the flow functions in the perturbed domains (appropriate estimations are executed in [3]).

2. It is obtained in [2] that roughnesses, not narrow but with characteristic dimensions (1.1) for $a \leqslant b \leqslant c$ on a flat plate, induce the pressure perturbation $\Delta p_1 \sim O(b^{2/3})$. If the roughnesses are submerged entirely in a shear, near-wall part of the unperturbed boundary layer on the plate ($u \sim y$), then for $a \sim b \sim O(\varepsilon^{3/2})$ their flow is described by Navier—Stokes equations and the pressure perturbation $\Delta p_2 \sim ku^2a \sim O(\varkappa\varepsilon^{5/2}) \ll \Delta p_1 \sim O(\varepsilon)$ is induced on the curved surface.

For the compensation flow regime around the roughness $a \sim O(\varepsilon b^{1/3})$, $\varepsilon^{3/2} < b < \varepsilon^{3/4}$, when there is no interaction with the external inviscid flow in a first approximation, the pressure perturbation $\Delta p_2 \sim ku\Delta ub \sim O(\varkappa b^{5/3}) \ll \Delta p_1 \sim O(b^{2/3})$ is induced on the curved surface for $b \leqslant \varepsilon$ or $\Delta p_2 \sim ku\Delta u\varepsilon \sim O(\varkappa b^{2/3}\varepsilon) \ll \Delta p_1 \sim O(b^{2/3})$ for $\varepsilon < b < \varepsilon^{3/4}$.

If the roughnesses perturb the external inviscid flow ($a \sim O(b^{5/3})$, $\varepsilon^{3/4} \leqslant b \leqslant \varepsilon^{3/5}$ are flows in the regime of free interaction or for a given pressure distribution), then the pressure perturbation $\Delta p_2 \sim ku\Delta ub \sim O(\varkappa b^{5/3}) \ll \Delta p_1 \sim O(b^{2/3})$ is induced on the curves surface.

Therefore, for all possible flow regimes of non-narrow ($a \leqslant b \leqslant c$) roughnesses (1.1) $\Delta p_2 \ll \Delta p_1$ for $\varkappa \leqslant 1$, and, consequently, the influence of the surface curvature will in no way be felt for them in a first approximation.

3. Narrow roughnesses with the characteristic dimensions (1.1) and for $a \leqslant c < b$ on a flat surface induce the pressure perturbation $\Delta p_1 \sim O(c^2/b^{4/3})$ [2]. If $a \sim O(\varepsilon b^{1/3})$, $\varepsilon b^{1/3} \leqslant c < \varepsilon/b^{1/3}$ The part ABD of the ABC plane in Fig. 2, the lines AB, BC, AC and BD correspond to $c \sim O(\varepsilon b^{1/3})$, $b \sim O(1)$, $b \sim c$ and $c \sim O(\varepsilon/b^{1/3})$), then the pressure perturbation is produced because of roughness interaction with the near-wall shear part of the unperturbed boundary layer on the flat surface and there is no interaction with the external inviscid flow. For $a \sim O(cb^{2/3})$, $\varepsilon/b^{1/3} < c \quad \varepsilon/b^{2/3}$ (the bed plane in Fig. 2, the line BE corresponds to $c \sim O(\varepsilon/b^{2/3})$), a pressure perturbation is produced because of roughness interaction with the external inviscid flow. If the characteristic roughness dimensions are incident on the intersection of the planes ABC and BED, the line BD, then the pressure perturbation is produced because of roughness interaction with the whole boundary layer on the flat surface. Perturbation transmission upstream is missing for all flow regimes of narrow roughnesses on a flat surface.

The pressure perturbation $\Delta p \sim \Delta p_1 \sim \Delta p_2 \sim O(c^2/b^{4/3})$ should be induced for narrow roughnesses on a curved surface in the near-wall domain 3 with characteristic dimensions $x \sim O(b)$, $y \sim O(a)$, $z \sim O(c)$, $a \sim O(\varepsilon b^{1/3})$. Then estimates are obtained for the velocity components $w \sim \Delta p^{1/2} \sim O(c/b^{2/3})$, $u \sim O(b^{1/3})$ and $v \sim O(\varepsilon/b^{1/3})$ from the conservation equations of the transverse momentum and the continuity. Consequently, the following independent variables and asymptotic expansions of the flow functions are introduced into domain 3 of the viscous nonlinear perturbations:

$$x = bx_3,\ y = \varepsilon b^{1/3}y_3,\ z = cz_3,\ u = b^{1/3}u_3 + \ldots,\ v = (\varepsilon/b^{1/3})v_3 + \ldots,\ w = (c/b^{2/3})w_3 + \ldots,\ \Delta p \approx (c^2/b^{4/3})p_3 + \ldots \tag{3.1}$$

The pressure perturbation $\Delta p_2 \sim ku^2a \sim O(\varkappa\varepsilon b)$ is induced on the curved surface in domain 3. Equating the orders of the perturbations Δp_1 and Δp_2 it is easy to see that $\Delta p_1 \sim \Delta p_2$ for $c \sim O(\varkappa^{1/2}\varepsilon^{1/2}b^{7/6})$ is the line FG in Fig. 2. For $c \sim a \sim O(\varepsilon b^{1/3})$

$$a \sim c \sim O(\varepsilon^{6/5}/\varkappa^{1/5}),\ b \sim O(\varepsilon^{3/5}/\varkappa^{3/5}) \tag{3.2}$$

is the point F in Fig. 2. Then substitution of the expansions (3.1) into the Navier—Stokes equations and completion of the passage to the limit as $\varepsilon \sim 0$ show that in a first

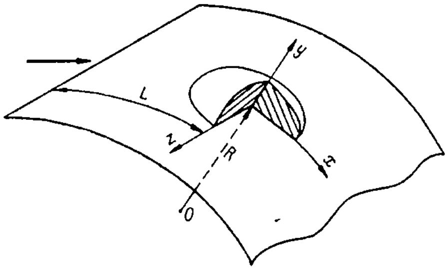

Fig. 1

approximation the flow around roughnesses with characteristic dimensions (3.2) on a curved surface is described by parabolized Navier—Stokes equations in the longitudinal direction

$$\begin{gathered}\frac{\partial u_3}{\partial x_3}+\frac{\partial v_3}{\partial y_3}+\frac{\partial w_3}{\partial z_3}=0,\quad u_3\frac{\partial u_3}{\partial x_3}+v_3\frac{\partial u_3}{\partial y_3}+w_3\frac{\partial u_3}{\partial z_3}=\frac{\partial^2 u_3}{\partial y_3^2}+\frac{\partial^2 u_3}{\partial z_3^2},\\ u_3\frac{\partial v_3}{\partial x_3}+v_3\frac{\partial v_3}{\partial y_3}+w_3\frac{\partial v_3}{\partial z_3}\mp Ku_3^2+\frac{\partial p_3}{\partial y_3}=\frac{\partial^2 v_3}{\partial y_3^2}+\frac{\partial^2 v_3}{\partial z_3^2},\\ u_3\frac{\partial w_3}{\partial x_3}+v_3\frac{\partial w_3}{\partial y_3}+w_3\frac{\partial w_3}{\partial z_3}+\frac{\partial p_3}{\partial z_3}=\frac{\partial^2 w_3}{\partial y_3^2}+\frac{\partial^2 w_3}{\partial z_3^2}.\end{gathered}\tag{3.3}$$

The usual adhesion and non-penetration conditions

$$u_3=v_3=w_3=0\ \ (y_3=f(x_3,\ z_3)).\tag{3.4}$$

should be satisfied on the surface of small roughness. The external boundary conditions are obtained from a merger with the solution for the near-wall shear part of the unperturbed boundary layer on a curved surface

$$u_3\to Ay_3,\ \ p_3\to\pm KA^2y_3^3/3,\ \ v_3,\ w_3\to 0\ \ \left(x_3^2+y_3^2+z_3^2\to\infty\right).\tag{3.5}$$

Here $A = (\partial u_{20}/\partial y_2)$ for $y_2 = 0$ ($y = \varepsilon y_2$, $u_{20}(y_2)$ is the profile of the longitudinal velocity component in the unperturbed boundary layer on the curved surface at a point where there is a small roughness).

The line FG ($c \sim O(\varkappa^{1/2}\varepsilon^{1/2}b^{7/6})$) in Fig. 2 is constructed for $\varkappa \sim O(1)$. For curved surface with $1 > \varkappa \geqslant \varepsilon$ the line FG and all subsequent constructions will evidently be shifted to the point B.

4. For thin narrow roughnesses with the characteristic dimensions

$$a\sim O(\varepsilon b^{1/3}),\ c\sim O(\varkappa^{1/2}\varepsilon^{1/2}b^{7/6}),\ (\varepsilon/\varkappa)^{3/5}<b\leqslant 1\tag{4.1}$$

(the line FG in Fig. 2), the flow in domain 3 is described in a first approximation by equations of the thin viscous layer type

$$\begin{gathered}\frac{\partial u_3}{\partial x_3}+\frac{\partial v_3}{\partial y_3}+\frac{\partial w_3}{\partial z_3}=0,\quad u_3\frac{\partial u_3}{\partial x_3}+v_3\frac{\partial u_3}{\partial y_3}+w_3\frac{\partial u_3}{\partial z_3}=\frac{\partial^2 u_3}{\partial y_3^2},\\ \frac{\partial p_3}{\partial y_3}=\pm Ku_3^2,\quad u_3\frac{\partial w_3}{\partial x_3}+v_3\frac{\partial w_3}{\partial y_3}+w_3\frac{\partial w_3}{\partial z_3}+\frac{\partial p_3}{\partial z_3}=\frac{\partial^2 w_3}{\partial y_3^2}.\end{gathered}\tag{4.2}$$

The solution of the system (4.2) should satisfy the inner (3.4) and initial boundary conditions

$$u_3\to Ay_3,\ \ p_3\to\pm KA^2y_3^3/3,\ v_3,\ w_3\to 0\ \ (x_3\to-\infty,\ z_3\to\pm\infty),$$

which are obtained from a merger with the solution for the near-wall shear part of the unperturbed boundary layer on a curved surface.

To find the external boundary conditions, it is necessary, in addition, to examine the domain 2 with thickness $y \sim O(c)$ for $\varepsilon^{6/5}/\varkappa^{1/5} < c < \varepsilon$ or $y \sim O(\varepsilon)$ for $\varepsilon \leqslant c \leqslant \varkappa^{1/2}\varepsilon^{1/2}$. Utilizing the relationship for $\Delta p_2 \sim ku\Delta uy$ and the continuity equation and the conservation of the transverse momentum equation we have in the first case

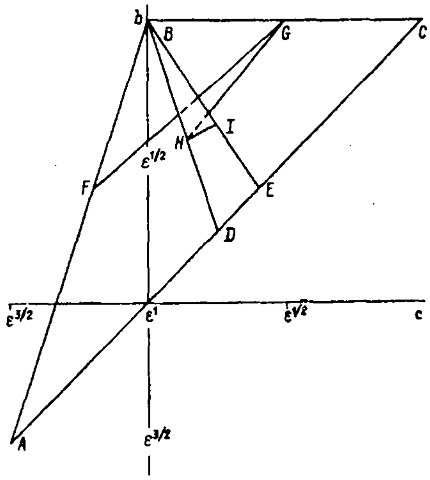

Fig. 2

$$u\sim O(c/\varepsilon),\ \Delta u\sim O(\varepsilon/\varkappa b^{4/3}),\ v\sim O(\varepsilon c/\varkappa b^{7/3}),\ w\sim O(\varepsilon/b^{1/3}), \tag{4.3}$$

and in the second

$$u\sim O(1),\ \Delta u\sim O(c^2/\varkappa\varepsilon b^{4/3}),\ v\sim O(c^2/\varkappa b^{7/3}),\ w\sim O(c/b^{1/3}). \tag{4.4}$$

These estimates show that in a first approximation the perturbations due to roughnesses with characteristic dimensions (4.1) does not penetrate into domain 2 (perturbations of the flow functions in domain 2 are less in orders of magnitude than in domain 3). Consequently, the solution of the system (4.2) should satisfy the external boundary conditions

$$u_3\to Ay_3,\quad p_3\to\pm KA^2y_3^3/3,\ v_3,\ w_3\to 0\quad (y_3\to\infty),$$

i.e., the external boundary conditions (3.5) are also valid for the case under consideration.

5. Flow regimes around small roughnesses on curved surfaces are examined in Secs. 3 and 4 when the pressure perturbation Δp_2 is induced in a domain 3 with characteristic thickness equal to the characteristic roughness thickness a in order of magnitude. The external domain 2 here remains unperturbed in a first approximation. Evidently the domain 2 was perturbed and the pressure perturbation Δp_2 was induced there, and an additional displacement thickness should be produced in domain 3. But necessary for this is the non-trivial merger of asymptotic expansions for the longitudinal velocity component u in domains 2 and 3 [2]. Comparing the asymptotic expansions of the flow functions (3.1) for the domain 3 and their estimates (4.3) and (4.4) in domain 2, it can be obtained that this condition is satisfied for $c\sim O(\varkappa^{1/2}\varepsilon^{1/2}b^{5/6})$, $\varepsilon\leqslant c\leqslant\varkappa^{1/2}\varepsilon^{1/2}$, the line HG in Fig. 2. Then the roughnesses with the characteristic dimensions

$$a\sim O(\varepsilon b^{1/3}),\ c\sim O(\varkappa^{1/2}\varepsilon^{1/2}b^{5/6}),\ (\varepsilon/\varkappa)^{3/5}\leqslant b\leqslant 1 \tag{5.1}$$

the flow in domain 3 is described in a first approximation by the Prandtl spatial boundary layer equations without the term $\partial p/\partial x$ in the equation for conservation of the longitudinal momentum

$$\begin{gathered}\frac{\partial u_3}{\partial x_3}+\frac{\partial v_3}{\partial y_3}+\frac{\partial w_3}{\partial z_3}=0,\quad u_3\frac{\partial u_3}{\partial x_3}+v_3\frac{\partial u_3}{\partial y_3}+w_3\frac{\partial u_3}{\partial z_3}=\frac{\partial^2u_3}{\partial y_3^2},\\ \frac{\partial p_3}{\partial y_3}=0,\quad u_3\frac{\partial w_3}{\partial x_3}+v_3\frac{\partial w_3}{\partial y_3}+w_3\frac{\partial w_3}{\partial z_3}+\frac{\partial p_3}{\partial z_3}=\frac{\partial^2w_3}{\partial y_3^2}.\end{gathered} \tag{5.2}$$

The solution of the system (5.2) should satisfy the inner (3.4) and initial boundary conditions

$$u_3\to Ay_3,\ v_3,\ w_3,\ p_3\to 0\ (x_3\to-\infty,\ z_3\to\pm\infty), \tag{5.3}$$

that are obtained from merging with the solution for the near-wall shear part of the unperturbed boundary layer on a curved surface.

New independent variables and asymptotic expansions of the flow functions

$$x = bx_2,\ y = \varepsilon y_2,\ z = cz_2,\ u = u_{20}(y_2) + (c^2/\varkappa\varepsilon b^{4/3})u_2 + \ldots,$$
$$v = (c^2/\varkappa b^{7/3})v_2 + \ldots, \tag{5.4}$$
$$w = (c/b^{1/3})\,w_2 + \ldots,\ \Delta p \approx \pm\varkappa\varepsilon K\int_0^{y_2} u_{20}^2 dy_2 + (c^2/b^{4/3})\,p_2 + \ldots$$

are introduced when using the estimates (4.4) in domain 2. Substitution of the expansions (5.4) into the Navier—Stokes equations and completion of the passage to the limit as $\varepsilon \to 0$ show that in a first approximation the flow around the roughnesses (5.1) in domain 2 is described by the system of linear inviscid equations

$$\frac{\partial u_2}{\partial x_2} + \frac{\partial v_2}{\partial y_2} = 0,\quad u_{20}\frac{\partial u_2}{\partial x_2} + v_2\frac{du_{20}}{dy_2} = 0,$$
$$\frac{\partial p_2}{\partial y_2} = \pm 2Ku_{20}u_2,\quad u_{20}\frac{\partial w_2}{\partial x_2} + \frac{\partial p_2}{\partial z_2} = 0,$$

that allows for partial integration

$$u_2 = Ddu_{20}/dy_2,\ v_2 = -u_{20}\partial D/\partial x_2,\ p_2 = \mp KD\left(1 - u_{20}^2\right) + p_2(y_2 \to \infty) \tag{5.5}$$

($D = D(x_2, z_2)$ is the displacement thickness of domain 3). Merging the asymptotic expansions of the flow functions (3.1) and (5.4) in the domains 2 and 3 when using the relationships (5.5) permits external boundary conditions to be obtained for the system (5.2)

$$u_3 \to A(y_3 + D),\ w_3 \to 0,\ p_3 = \mp KD + p_2(y_2 \to \infty)\ (y_3 \to \infty). \tag{5.6}$$

To determine the function p_2 (x_2, $y_2 \to \infty$, z_2) it is necessary, in addition, to examine the perturbed domain 1 of inviscid external flow with the characteristic dimensions $x \sim O(b)$, $y \sim z \sim O(c)$, in which new variables and asymptotic expansions of the flow functions are introduced

$$x = bx_1,\ y = cy_1,\ z = cz_1,\ u \approx 1 + b^{2/3}u_1 + \ldots, \tag{5.7}$$
$$v = (c/b^{1/3})v_1 + \ldots,\ w = (c/b^{1/3})w_1 + \ldots,\ \Delta p \approx \varkappa cKy_1 + (c^2/b^{4/3})p_1 + \ldots$$

Merger of the asymptotic expansions of the flow functions (5.4) and (5.7) in domains 2 and 1 shows that for $b > (\varepsilon/\varkappa)^{3/7}$, $c \sim O(\varkappa^{1/2}\varepsilon^{1/2}b^{5/6})$ the vertical velocity component v is greater in order of magnitude in domain 1 than in domain 2. This means that for $b > (\varepsilon/\varkappa)^{3/7}$ for the roughnesses (5.1) the domain 1 remains unperturbed and all the perturbations due to a small roughness should damp out in domain 2, i.e., $p_2(x_2,\ y_2 \to \infty,\ z_2) = 0$.

If

$$a \sim O(\varepsilon^{8/7}/\varkappa^{1/7}),\ b \sim O(\varepsilon^{3/7}/\varkappa^{3/7}),\ c \sim O(\varkappa^{1/7}\varepsilon^{6/7}) \tag{5.8}$$

(the point H in Fig. 2), then substitution of the expansions (5.7) into the Navier—Stokes equations and completion of the passage to the limit as $\varepsilon \to 0$ show that in a first approximation the flow in domain 1 is described by a system of linear inviscid equations that it is convenient to reduce to one equation for the pressure perturbation

$$\partial^2 p_1/\partial y_1^2 + \partial^2 p_1/\partial z_1^2 = 0,\ p_1 \to 0\quad (x_1^2 + y_1^2 + z_1^2 \to \infty), \tag{5.9}$$
$$p_1(x_1,\ 0,\ z_1) = p_2(x_2,\ y_2 \to \infty,\ z_2).$$

The inner boundary condition for this equation is obtained from merger of the expansions (5.4) and (5.7) when using the relationships (5.5)

$$\partial p_1/\partial y_1 = \partial^2 D/\partial x_1^2\quad (y_1 = 0). \tag{5.10}$$

For $b < (\varepsilon/\varkappa)^{3/7}$, $c \sim O(\varkappa^{1/2}\varepsilon^{1/2}b^{5/6})$ infinitely large perturbations of the vertical velocity component v are induced in domain 1, which is inadmissible.

The flow regimes around roughnesses examined in this section were investigated in [3, 4]; the results of numerical computations of the boundary value problem (3.4), (5.2), (5.3), and (5.6) are represented in [4] for $p_2(x_2,\ y_2 \to \infty,\ z_2) = 0$ in a linear approximation.

6. The characteristic thickness of "thick" roughnesses (in the terminology of [2]) $a \sim O(cb^{2/3})$ (the BED plane in Fig. 2) is greater in order of magnitude than the thickness of the viscous layer $\delta_1 \sim O(\varepsilon b^{1/3})$ near their surface. Consequently, the asymptotic expansions (3.1) are valid in domain 3 except that the coordinate y_3 is now measured from the roughness surface $f(x_3, z_3)$. The vertical velocity component v in domain 2 should equal the characteristic slope of the roughness in the longitudinal direction $v \sim a/b \sim O(c/b^{1/3})$ in order of magnitude. Using the asymptotic expansions (5.4) it is easy to obtain that this condition is satisfied for $c \sim O(\varkappa b^2)$ (the line HI in the plane BED in Fig. 2). Then the flow around roughnesses with characteristic dimensions

$$a \sim O(cb^{2/3}),\ c \sim O(\varkappa b^2),\ (\varepsilon/\varkappa)^{3/7} < b < (\varepsilon/\varkappa)^{3/8} \tag{6.1}$$

is described in a first approximation as $\varepsilon \to 0$ by the system (5.2) whose solution should satisfy the initial boundary (5.3) and the inner boundary conditions

$$u_3 = v_3 = w_3 = 0\ (y_3 = 0). \tag{6.2}$$

New independent variables are introduced in domain 2

$$x = bx_2,\ y = \varepsilon y_2 + cb^{2/3}f(x_2, z_2) + \ldots,\ z = cz_2. \tag{6.3}$$

Application of the variables (6.3) and the asymptotic expansions of the flow functions (5.4) shows that in a first approximation as $\varepsilon \to 0$ in domain 2 equations of the following form are valid

$$\frac{\partial u_2}{\partial x_2} - \frac{\partial f}{\partial x_2}\frac{du_{20}}{dy_2} + \frac{\partial v_2}{\partial y_2} = 0,\quad u_{20}\left(\frac{\partial u_2}{\partial x_2} - \frac{\partial f}{\partial x_2}\frac{du_{20}}{dy_2}\right) + v_2\frac{du_{20}}{dy_2} = 0,$$
$$\frac{\partial p_2}{\partial y_2} = \pm 2Ku_{20}u_2,\quad u_{20}\frac{\partial w_2}{\partial x_2} + \frac{\partial p_2}{\partial z_2} = 0,$$

which allow the partial integration

$$u_2 = D\frac{du_{20}}{dy_2},\ v_2 = u_{20}\left(\frac{\partial f}{\partial x_2} - \frac{\partial D}{\partial x_2}\right),\ p_2 = \mp KD(1 - u_{20}^2) + p_2(y_2 \to \infty). \tag{6.4}$$

Merger of the asymptotic expansion of the flow functions (3.1) and (5.4) (with the independent variables (6.3)) in domains 3 and 2 when using the relationships (6.4) show that $D(x_2, z_2) = -f(x_2, z_2)$ in the case under consideration and the solution of the system (5.2) satisfies the external boundary conditions

$$u_3 \to Ay_3,\ w_3 \to 0,\ p_3 = \pm Kf + p_2(y_2 \to \infty)\ (y_3 \to \infty). \tag{6.5}$$

The unknown function $p_2(x_2, y_2 \to \infty, z_2)$ is determined from the solution (5.9) for domain 1 and the internal boundary condition

$$\partial p_1/\partial y_1 = -2\partial^2 f/\partial x_1^2\quad (y_1 = 0). \tag{6.6}$$

7. The analysis performed for the flow regimes around narrow roughnesses on a curved surface shows the essential distinctions in the mechanism of perturbation origin as compared with the flow around roughnesses on a flat surface (see Sec. 3 and [2]). Thus, the roughnesses (3.2) and (4.1) that induce the greatest viscous layer in thickness near their surfaces (in domain 3) do not perturb the external domains 2 and 1 (the line FG in Fig. 2) in a first approximation. The perturbations due to roughnesses are here equilibrated in the domain 3 by a strong centrifugal force field $\Delta p_2 \sim y^3$, the flows are described by Navier—Stokes equations parabolized in the longitudinal direction or by equations of the thin viscous layer type (boundary value problems (3.3)-(3.5) or (3.4), (3.5), and (4.2), respectively).

For the roughnesses (5.1) (the line HG in Fig. 2), the centrifugal forces do not already equilibrate the perturbations due to roughnesses in the thinner layer 3. This occurs on the fundamental boundary layer thickness on the curved surface in domain 2 and the flows are described by the Prandtl spatial boundary layer equations with a "centrifugal" interaction condition between domains 3 and 2 (the boundary value problem (3.4), (5.2), (5.3), and (5.6) for $p_2(x_2, y_2 \to \infty, z_2) = 0$). For the roughnesses (5.8) (the point H in Fig. 2), the perturbations do not damp out in domain 2 and penetrate into domain 1 (here it is necessary to solve the boundary value problem (5.9) and (5.10) in addition in order to determine the function $p_2(x_2, y_2 \to \infty, z_2)$.

The thinnest of all the viscous layers examined above is induced on their surfaces for the flow around the roughnesses (6.1) ("thick" in the terminology of [2]). Hence, domains 2 and 1 are perturbed only by the shape of the roughness, and in this case the Prandtl spatial boundary layer equations with a given pressure distribution must be solved (the boundary value problem (5.2), (5.3), (5.9), (6.2), (6.5), and (6.6), the line HI in Fig. 2).

It is obtained in [2] that because of the diminution of the transverse velocity component w in order of magnitude during passage from the flow around non-narrow roughnesses ($c \sim b$) on a flat plate to narrow ($c < b$) the perturbation transmission upstream disappears for them. If the roughness is on a curved surface, then Δp_1 and Δp_2 have identical signs for a concave surface and different signs for a convex. Consequently, for narrow roughnesses on a concave surface the total pressure perturbation is greater than for roughnesses on a flat surface. This results in an increase in the transverse velocity component w and the origination of upstream perturbation transmission [3, 4]. For roughnesses on convex surfaces such a phenomenon should not be realized.

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STEADY SURFACING OF A SINGLE BUBBLE IN AN INFINITE VOLUME OF LIQUID

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The motion of individual gas bubbles has long been an object of investigation. Important theoretical solutions have been obtained and a sizable body of experimental data has been accumulated. Recent years have seen the broad use of numerical methods to solve the Navier—Stokes equations with an unknown boundary in regard to the study of bubble motion [1, 2].

Here, we use numerical solutions that we obtained to the complete Navier—Stokes equations and the results of an experiment to analyze the simultaneous effect of the viscosity of the liquid and surface tension on the rate of surfacing and form of individual bubbles. We also determine the limits of disturbance of the sphericity of gas bubbles and the formation of eddies in the rear part of the bubbles.

We examine the steady surfacing of an axisymmetric bubble with the boundary Γ. The volume of the bubble is constant, as is the pressure inside it. We introduce cartesian coordinate system x_1, x_2, x_3, connected with the center of the bubble. The x_3 axis is directed along the upward velocity of the gas cavity, **u**, **n** is the unit vector of an outward normal to Γ, τ is the unit vector of a tangent to Γ. The motion of the viscous liquid outside the closed surface Γ is described by the system of equations

$$(\mathbf{v}\nabla)\mathbf{v} + \nabla p/\rho = \nu\Delta\mathbf{v}, \ \nabla\mathbf{v} = 0, \tag{1}$$

where p is the modified pressure function: $p = q + \rho g x_3 - p_0$; q is the pressure in the liquid; p_0 is the pressure at the level $x_3 = 0$; g is acceleration due to gravity; ρ is the density of the liquid; ν is the kinematic viscosity of the liquid.

The following conditions are satisfied on the free boundary Γ: impermeability

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